A WIDE-FIELD CORRECTOR AT THE PRIME FOCUS OF A RITCHEY-CHRÉTIEN TELESCOPE*

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Abstract — We propose a form of a lens corrector at the prime focus of a hyperboloidal mirror that provides a flat field of view up to 3° diameter at image quality $D_{80} < 0.8$ arcsec in integrated (0.32–1.10 μ m) light. The corrector consists of five lenses made of fused silica. Only spherical surfaces are used, so the system is capable of achieving better images, if necessary, by aspherizing the surfaces. The optical system of the corrector is stable in the sense that its principal features are retained when optimized after significant perturbations of its parameters. As an example, three versions of the corrector are designed for the V. M. Blanco 4-m telescope at Cerro Tololo Inter-American Observatory with 2°.12, 2°.4, and 3°.0 fields of view. © 2004 MAIK "Nauka/Interperiodica".

 $\textit{Key words}\colon$ astronomical observing techniques, devices and instruments.

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Introduction

The advent of reflectors with aperture diameters of 8-10 m required a revision of observational programs for telescopes of preceding generations. Emphasis was placed on designing systems of adaptive optics and realization sky surveys at the prime focus with a wide-field corrector. The choice of the second way is determined by several factors.

First of all, a few observational programs aimed at solving important astrophysical problems, in particular, at studying gamma-ray bursts, searching for hidden mass, and analyzing gravitational lensing in clusters of galaxies, are of current interest. For obvious reasons, the diameter of a Schmidt telescope is difficult for increasing up to values well above the current level of ~ 1.3 m. Special 4-m telescopes with lens correctors designed together with the primary mirror are being proposed to solve these problems: the Next Generation Lowell Telescope (NGLT) (Blanco et al. 2002) and the Visible and Infrared Survey Telescope for Astronomy (VISTA) (McPherson et al. 2002; Emerson and Sutherland 2002). Particular attention is given to the Large Synoptic Survey Telescope (LSST) with an effective aperture of about 6.5 m at a primary mirror diameter of 8.4 m (Angel et al. 2000; Tyson 2002; Seppala 2002). At the same time, being equipped with the prime-focus correctors with a field of $\sim 1^{\circ}.5 - 2^{\circ}.0$ diameter, the existing 4-m Ritchey-Chrétien telescopes achieve an efficiency comparable to the efficiency for the telescopes being designed.

Secondly, the fact that a field corrector to an existing telescope can be made relatively fast also seems important.

Finally, at a diameter of ~ 4 m and a focal ratio of $\sim 2.5-3$, the primary mirror of a Ritchey–Chrétien telescope with a roughly afocal field corrector can be matched in modulation-transfer function with the main modern CCD detectors with pixel sizes of $\sim 15~\mu m$. Thus, the challenging problem during observations at the Cassegrain focus is solved in a natural way.

Much effort has been made to design field correctors at the prime focus of a reflector (see the reviews by Wynne 1972; Mikhelson 1976, §7.5; Wilson 1996, §4.3; Schroeder 2000, §9.2). The correctors designed by Ross (1935) and Wynne (1968) were the systems that determined the development of this area of research for a long time. The former corrector has a flat field of view of $2w \simeq 15'$ diameter with stellar images better than 1"; the latter detector has an about 50' field of similar quality. Many of the modern reflectors are equipped with three-lens Wynne correctors or modifications of this system.

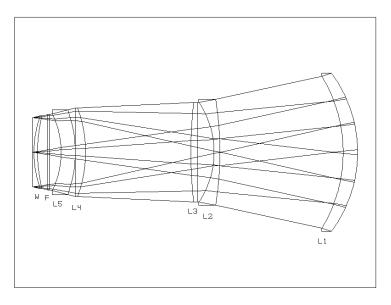


Figure 1: * Corrector "S". The last elements are filter (F) and detector window (W).

The observational programs being planned require a field of view no less than 1°.5 in diameter. Thus, for example, the NGLT and VISTA projects mentioned above are to provide a flat field of 2° diameter. The correctors designed for these telescopes have four or five aspherical lenses, with the diameter of the front lens reaching 1.25 m. The front lens of the LSST corrector is of 1.34 m diameter; the concave surfaces of the lenses are eighth-order aspherics.

Here, we propose a new type of corrector at the prime focus of a hyperboloidal mirror (Fig. 1) that provides a flat field up to 3° in diameter at image quality¹ better than 0″.8 in integrated (0.32–1.10 μ m) light. Since observations are generally carried out in relatively narrow spectral ranges rather than in integrated light, the corresponding image size with the corrector is smaller than this value. Only spherical surfaces were used, so the system is not only relatively easy to make, but also is capable of achieving better images by aspherizing the surfaces. All lenses are made of fused silica. This material has good manufacturing properties and provides high transparency in the ultraviolet, which is often a key factor.

¹The diameter of a circle within which 80% of energy in the image of a point-like source is enclosed, D_{80} , is meant.

Table 1. General characteristics of the correctors

Parameter	Corrector			
	"R"	"S"	"T"	
Angular field of view, $2w$, deg	2.12	2.4	3.0	
Effective focal length with the telescope, mm	11 506.7	11 400.4	11 505.9	
Focal ratio	2.92	2.90	2.92	
Scale, $\mu m/arcsec$	55.79	55.27	55.78	
Linear field of view, mm	427	481	606	
Spectral range, μm	0.32 – 1.10	0.32 – 1.10	0.32 – 1.10	
Variation of image RMS-radius over field, 0.32–1.10 μ m	$13.2 – 15.6 \mu m$ 0''.24 – 0''.28	$12.4-15.6 \ \mu \text{m}$ $0''.22-0''.28$	$14.3 – 19.8 \ \mu \text{m}$ $0^{''}.26 – 0^{''}.35$	
Variation of D_{80} over field (center-edge, 0.32–1.10 μ m)	$33.2 – 38.5 \mu m$ 0''.60 – 0''.70	$31.8-38.0 \ \mu \text{m}$ $0''.58-0''.68$	$36.0 – 45.0 \ \mu \text{m}$ $0^{''}.64 – 0^{''}.80$	
Variation of D_{80} over field in 0.35–0.45 $\mu \mathrm{m}$ band	$20.0 – 39.3 \ \mu \text{m}$ $0^{"}.36 – 0^{"}.70$	$17.4-40.2 \ \mu \text{m}$ $0''.32-0''.72$	$19.6 – 52.8 \ \mu\mathrm{m} \\ 0^{''}.36 - 0^{''}.94$	
Variation of D_{80} over field in 0.54–0.66 μm band	$24.4 - 30.3 \ \mu \text{m}$ $0^{''}.44 - 0^{''}.54$	$20.2-25.8 \ \mu \text{m}$ $0^{"}.37 - 0^{"}.47$	$24.0 – 28.2 \mu m$ $0^{''}.44 – 0^{''}.50$	
Variation of D_{80} over field in 0.70–0.90 μm band	$25.8 – 38.3 \ \mu \text{m}$ $0^{"}.46 – 0^{"}.69$	$20.4-33.8 \ \mu \text{m}$ $0''.37 - 0''.61$	$25.4 – 38.8 \ \mu \text{m}$ $0^{''}.46 – 0^{''}.70$	
Transmittance (including reflections, without coatings, 0.32–1.10 μ m)	0.53-0.55	0.53-0.55	0.53 – 0.55	
Maximum distortion	0.42%	0.60%	0.61%	
Maximum gradient of distortion with wavelength, μm^{-1}	3.88×10^{-4}	2.25×10^{-4}	3.25×10^{-4}	
Types of lens surfaces	All spheres	All spheres	All spheres	
Maximum clear aperture, mm	900	1100	1300	

The proposed corrector system is designed for a hyperboloidal mirror with a conic constant typical of Ritchey-Chrétien telescopes. As specific examples, we discuss three versions of the corrector for the V. M. Blanco 4-m telescope at Cerro Tololo Inter-American Observatory — systems "R", "S", and "T", with 2°.12, 2°.4, and 3°.0 fields of view, respectively (Table 1). The last two systems should be considered to be basic, while in system "R" designed for

a reduced size of the front lens, we had to introduce noticeable distortions.

Primary mirror of the telescope

The parameters of the primary mirror of the Blanco telescope (Table 2) were taken from the report by Gregory and Boccas (2000). The central obscuration is produced by a hole in the mirror and stray-light baffles. Since images far from the diffraction limit are dealt with in wide-field observations, the central obscuration affects the images only slightly.

Table 2. Primary mirror

Parameter	Value		
Radius of curvature at vertex	$-21311.6\;\mathrm{mm}$		
Conic constant k	-1.09763		
Aperture diameter	$3934~\mathrm{mm}$		
Central obscuration	$1651~\mathrm{mm}$		

Note that the correctors described below need to be adjusted only slightly for a moderate variation of the parameters of the primary mirror given in Table 2. In particular, this is true for a paraboloidal primary mirror.

System "S"

The layout of corrector "S" is shown in Fig. 1 (for detailed information, see Table 3). The letters FS denote fused silica. The most commonly used Schott BK7 glass was taken as the material for the filter; clearly, the system is not critical in this regard, and choosing a different glass as well as adopting a different thickness of the filter can be easily compensated.

If we remove lenses L3 and L5 from system "S", then the remaining part will resemble the classical system by Wynne (1968). Necessity of addition of these two lenses to produce a really large field of view is caused by the fact that, in this case, doublets L2+L3 and L4+L5 are formed; each of them effectively suppresses the aberrations of the primary mirror, first of all, coma. It is interesting to note, in this connection, that the lens L3 has already appeared in the corrector designed by Delabre (2002) for a $2w = 0^{\circ}.95$ field

of view. Delabre's system consists of three lenses and a detector window which has an optical power.

Table 3. Design data for the system "S"

Surface	Comments	Radius of curvature,	Thickness,	Glass	Clear aperture,
number		mm	mm		mm
1 2	Aperture stop Primary mirror	∞	90.755		3934.00
	(k = -1.09763)	-21311.6	-8521.90	Mirror	3934.00
3	L1	-921.47	-100.00	FS	1100.00
4		-1017.83	-855.28		1056.78
5	L2	-2321.62	-40.00	FS	740.03
6		-620.63	-116.04		693.91
7	L3	∞	-45.00	FS	693.96
8		3077.69	-730.57		694.04
9	L4	-872.53	-70.13	FS	619.93
10		33728.22	-1.00		616.39
11	L5	-865.49	-98.20	FS	591.30
12		-620.76	-80.19		533.31
13	Filter	∞	-12.00	BK7	526.08
14		∞	-71.36	_	523.30
15	Window	1047.66	-25.00	FS	508.85
16		734.15	-10.00	_	508.06
17	Detector	∞			480.55

An optical power is also planned to be imparted to the detector window in the correctors described here. A slightly worse, but comparable image quality is achieved for a flat window. However, it seems natural to use additional degrees of freedom, given the total number of optical surfaces.

The five-lens system shown in Fig. 1 is *stable* in the sense that its principal features are retained when optimized after significant perturbations of its parameters. The final state in stable systems is reached abruptly; i.e., either a global or a nearly global minimum of the merit function is realized in the multi-dimensional space of optical parameters. The numerous variations of a three-lens corrector show that a similar stability is also characteristic of Wynne's triplet, but in lower-dimension space. These features of the five-lens system allow it to be considered as a new type of field corrector at the prime focus of a reflector.

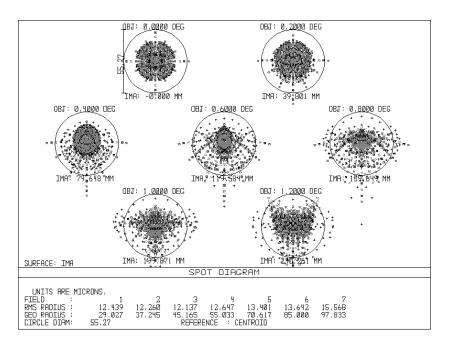


Figure 2: *

Spot diagram for corrector "S" over the range $0.32-1.10~\mu\mathrm{m}$ for field angles of 0, 0°.2, 0°.4, 0°.6, 0°.8, 1°.0, and 1°.2. The circle diameters correspond to 1 arcsec (55.27 $\mu\mathrm{m}$). The root-mean-square (RMS) and geometrical (GEO) radii of the images of a point-like source (in $\mu\mathrm{m}$) are indicated for each of seven field angles.

Figure 2 shows the spot diagram² for system "S" in integrated light. A slightly clearer idea of the image quality can be got from the plot of the fraction of enclosed energy in the diffraction stellar image shown in Fig. 3. We took special measures in order that the diameters of the stellar images remain constant over the entire field. As one can see from Fig. 3, D_{80} changes from 0".58 on the optical axis to 0".68 at the edge of the field. The image quality in narrow spectral bands is given in Table 1. Note that the refocusing range when passing from one spectral band to another is only a few hundredths of a millimeter; such a small value is attributable to the optimization of the system in integrated light.

The corrector is close to an afocal system, so the focal length of the telescope exceeds the focal length of the primary mirror only slightly (see

²The distribution of light rays in a stellar image on the focal plane.

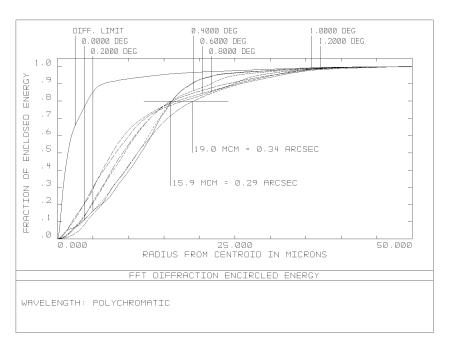


Figure 3: *

Integral energy distribution along the radius in the diffraction stellar image for corrector "S" in the range 0.32–1.10 μ m for field angles of 0, 0°.2, 0°.4, 0°.6, 0°.8, 1°.0, and 1°.2. The 80% level and the corresponding extreme values of the radius (in μ m and arcseconds) are indicated.

Tables 1 and 2). At a pixel size of $\sim 15 \ \mu \text{m}$ characteristic of the CCD detectors currently used in astronomy, an angle of 0".27 corresponds to one pixel in the focal plane. Thus, approximately 1.5 to 2.5 pixels fit into the diameter D_{80} , depending on the width of the spectral range used in observations. This matching of the optical system with the detector should be considered to be satisfactory.

Since the lenses of the corrector are made of fused silica, its transmittance depends weakly on wavelength in the spectral range 0.32–1.10 μ m considered here (the data of Table 1 refer to lenses L1–L5). The deposition of effective modern coatings will provide a $\sim 83\%$ transmittance of the corrector. Thus, addition of two fused silica lenses to Wynne's corrector, which will affect the system transparency only slightly, allows us to significantly expand the field of view with good images.

For the linear fields of view of the order half a meter of interest, not

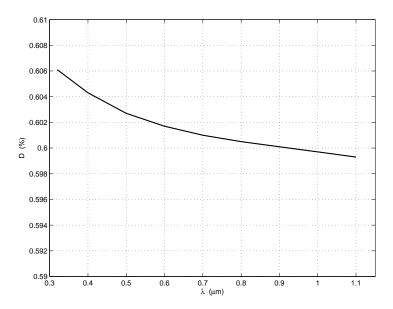


Figure 4: * Relative image distortion versus wavelength at the edge of the field of system "S".

so much the image distortion as its variation with wavelength is hazardous (Ingerson 1997). In system "S", the positive³ distortion reaches its maximum at the edge of the field of view in the ultraviolet; the exact value is 0.606% for a field angle of $w=1^{\circ}.2$ and a wavelength of $0.32~\mu m$. If we do not pose a special astrometric problem, then this value may be considered negligible. Otherwise, being constant with time, the distortion can be taken into account when processing the data.

In our case, the variation of the distortion with wavelength is shown in Fig. 4. The maximum (in absolute value) distortion gradient is $-2.25 \times 10^{-4} \ \mu \text{m}^{-1}$. Suppose, for example, that the observations are carried out in a 0.1- μ m-wide ultraviolet band. The length of the spectrum attributable to the distortion variation with wavelength is then 5.4 μ m (the radius of the field of view was taken to be 240 mm). This length is only a small fraction of the total image size $D_{80} \simeq 40 \ \mu$ m at the edge of the field in the ultraviolet. Since the effect under discussion plays an appreciably lesser role in other spectral ranges, the distortion in system "S" may be considered acceptable.

³Often called *pincushion*.

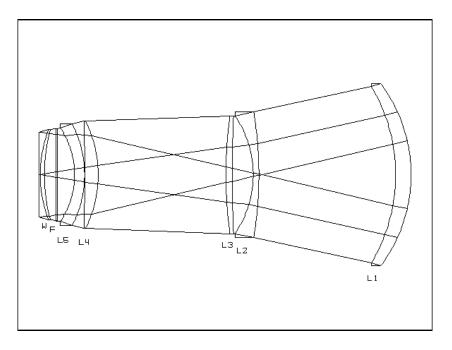


Figure 5: * Corrector "T". The last elements are filter (F) and detector window (W).

System "T"

The layout of corrector "T" is shown in Fig. 5; its parameters are given in Table. 4.

In this case, the image quality (Fig. 6) is only slightly worse than that for system "S". As above, an angle of 0".27 corresponds to a 15- μ m detector pixel in the focal plane. Therefore, the above remarks concerning the matching of the optical system and the light detector remain valid.

As far as the basic optical system remained virtually unchanged when the field of view increased significantly, from 2°.4 up to 3°.0, it primarily corresponds just to wide-field observations. In fact, the introduction of two doublets effectively suppressing coma pursued this goal. It is still possible to find a four-lens corrector with an image quality that is only slightly worse than that for the five-lens system "S", but we failed to find a four-lens "double" of system "T" with its 3° field.

The refocusing range of the corrector when changing the spectral band does not exceed 0.05 mm.

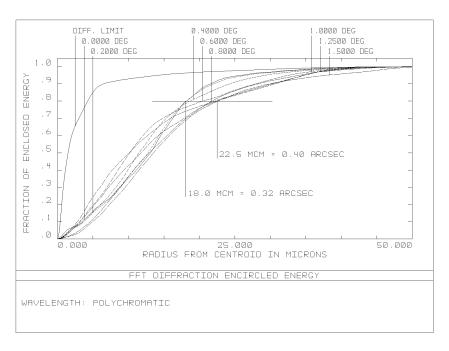


Figure 6: *

Integral energy distribution along the radius in the diffraction stellar image for corrector "T" in the range 0.32–1.10 μ m for field angles of 0, 0°.2, 0°.4, 0°.6, 0°.8, 1°.0, 1°.25, and 1°.5. The 80% level and the corresponding extreme values of the radius (in μ m and arcseconds) are indicated.

The transparency of the system under consideration in the entire spectral range 0.32–1.10 μm is virtually the same as that for system "S".

The type of image distortion (positive) was also preserved. At the edge of the field of view, the distortion slightly increases from 0.607% in the ultraviolet to 0.611% in the infrared. The distortion in both correctors is small; if necessary, an orthoscopic corrector can be designed rigorously. The largest distortion gradient with wavelength, namely, $3.25 \times 10^{-4} \ \mu \text{m}^{-1}$, is reached at the edge of the field again for $\lambda = 0.32 \ \mu \text{m}$. At observations in a 0.1- μ m-wide ultraviolet band, the distortion variation with wavelength causes the images to blur at the edge of the field by slightly less than 10 μ m. This value is almost twice as large as the value for system "S". However, as above, it is small compared to the sizes of the images themselves.

Table 4. Design data for the system "T"

		Radius			Clear
Surface	Comments	of curvature,	Thickness,	Glass	aperture,
number		mm	mm		$_{ m mm}$
1	Aperture stop	∞	90.755		3934.00
2	Primary mirror				
	(k = -1.09763)	-21311.6	-8150.16	Mirror	3934.00
3	L1	-1084.73	-110.00	FS	1300.00
4		-1226.17	-975.325		1256.42
5	L2	-2754.95	-40.00	FS	895.21
6		-742.97	-150.24		841.71
7	L3	9276.47	-45.00	FS	841.83
8		3011.10	-910.86		842.74
9	L4	-866.96	-96.00	FS	772.69
10		-14737.23	-1.00		769.07
11	L5	-792.85	-71.00	FS	726.26
12		-602.28	-121.60		669.50
13	Filter	∞	-12.00	BK7	661.82
14		∞	-81.44	_	658.87
15	Window	1269.90	-30.00	FS	643.51
16		901.66	-10.00		642.55
17	Detector	∞			606.38

System "R"

In the Introduction, we noted that decreasing the diameter of the front lens in system "R" entails a noticeable distortion of the basis system. For this reason, system "R" is given here as a supplement to the correctors considered above. Nevertheless, system "R", taken in itself, is of interest in realizing a field of view slightly larger than 2° (the adopted specific field diameter of 2°.12 corresponds to the diagonal of a square with a 1°.5 side).

Table 1 gives a description of system "R" enough to get an idea of the image quality. Table 5 lists parameters of the optical elements; further information can be obtained after inputting the data of Table 5 to some optical program.

As we see from Table 1, the relatively compact system "R" provides roughly the same image quality as does system "S", but within a somewhat smaller field of view.

Table 5. Design data for the system "R"

		Radius			Clear
Surface	Comments	of curvature,	Thickness,	Glass	aperture,
number		mm	mm		$_{ m mm}$
1	Aperture stop	∞	90.755		3934.00
2	Primary mirror				
	(k = -1.09763)	-21311.6	-9011.089	Mirror	3934.00
3	L1	-705.30	-81.34	FS	900.00
4		-823.90	-497.65		869.64
5	L2	-1483.06	-35.16	FS	670.60
6		-506.93	-304.37		621.38
7	L3	14674.93	-42.38	FS	614.17
8		1829.42	-472.97		613.92
9	L4	-385.00	-35.00	FS	512.86
10		-382.33	-51.69		494.85
11	L5	-745.23	-62.00	FS	494.24
12		-1168.35	-44.72		478.18
13	Filter	∞	-12.00	BK7	471.52
14		∞	-70.04	_	468.96
15	Window	878.97	-25.22	FS	455.64
16		596.19	-10.00	_	455.21
17	Detector	∞			427.71

Concluding remarks

The field correctors considered here are relatively simple systems: the lens surfaces are spherical, and the lenses themselves are made of the same material. The question as to whether to aspherize some or all of the surfaces should probably be solved depending on specific circumstances that include the need for achieving better images, the corrector production cost, etc.

Just as it occurs in adaptive optics systems, the requirement of providing high transmittance of the system in the outermost ultraviolet range $0.32-0.34~\mu m$ presents the greatest difficulty in using the lenses (see, in particular, Tokovinin et al. 2003). Since the total thickness of the corrector lenses is large, this requirement, in fact, narrows down the choice to one material — fused silica. Quartz optics is known to be transparent far beyond the range $0.32-1.10~\mu m$ considered here. From the optical point of view, it becomes possible to use a single material in such a complex system as the field corrector, because a moderate change in the focal length of the telescope is

admissible.

Apart from high transparency, there are also other reasons for seeking to make the system purely of fused silica:

- reliable manufacturing procedures for producing large homogeneous blanks of this material have now been developed;
- fused silica is well polished;
- it firmly holds coatings;
- all lenses of the system have not only a small, but also the same thermal expansion coefficient⁴;
- fused silica has a good time stability.

A discussion of attendant questions can be found in §3.3 of the monograph by Wilson (1999).

Observations with telescopes that provide subarcsecond images need to be corrected for differential atmospheric refraction (Wynne and Worswick 1986; Wynne 1986; Wilson 1996, §4.4; Schroeder 2000, §9.5). The corresponding atmospheric dispersion corrector (ADC) can be realized by making the lenses of the field corrector more complex. In recent years, however, an ADC has been customarily built into the field corrector as an independent device.

At first glance, systems of the type described here have a linear field of view that is too large for CCD detectors to be effectively used. Thus, for example, the diameter of the field of view is 481 mm for system "S" and exceeds 600 mm for system "T". Meanwhile, such field sizes are typical in the modern projects of wide-field telescopes. For example, a linear field of 550 mm diameter is expected to be achieved in the LSST project. The main difficulty here is associated not with the covering of the focal plane with many CCD chips, but with the necessity of rapidly reading out and promptly processing an extremely large amount of information. This problem has already been solved in some of the existing systems (Lesser and Tyson 2002; Walker 2002).

⁴According to Schott Lithotec (2003), it is 0.5×10^{-6} /K in the range $25^{\circ} - 100^{\circ}$ C.

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References

- [1] J.R.P.Angel, M.Lesser, R.Sarlot, and T.Dunham, ASP Conf. Ser. **195**, 81 (2000).
- [2] D.R.Blanco, G.Pentland, C.H.Smith, T.Dunham, and R.L.Millis, Proc. SPIE No. 4842-20 (2002).
- [3] B.Delabre, Optical Design for Astronomical Instruments (Rio de Janeiro), http://www.on.br/institucional/portuguese/ciclo2002/pub/ Delabre/ RIO2002b.PPT (2002).
- [4] J.P.Emerson and W.Sutherland, Proc. SPIE No. 4836-08 (2002).
- [5] B.Gregory and M.Boccas, *The Blanco 4-m Telescope*, http://www.ctio.noao.edu/telescopes/4m/4m.html (2000).
- [6] T.E.Ingerson, Empirical and Theoretical Modeling of the PFADC Corrector on the Blanco 4-m Telescope, http://www.ctio.noao.edu/telescopes/4m/pfadc/pfadc_tei.html (1997).
- [7] M.P.Lesser and J.A.Tyson, Proc. SPIE No. 4836-38 (2002).
- [8] Schott Lithotec, Synthetic Fused Silica. Optical and Technical Grades (2003).
- [9] A.McPherson, S.C.Craig, and W.Sutherland, Proc. SPIE No. 4837-10 (2002).
- [10] N.N.Mikhelson, Optical Telescopes. Theory and Design, Moscow, Nauka, (1976) [in Russian].
- [11] F.E.Ross, Astrophys. J. 81, 156 (1935).
- [12] L.G.Seppala, Proc. SPIE No. 4836-19 (2002).

- [13] D.J.Schroeder, Astronomical Optics, Academic, San Diego (2000).
- [14] A.Tokovinin, B.Gregory, H.E.Schwarz, V.Terebizh, S.Thomas, Proc. SPIE 439, 673 (2003).
- [15] J.A.Tyson, Proc. SPIE No. 4836-04 (2002).
- [16] A.R. Walker, Mem. Soc. Astron. Ital. 73, 23 (2002).
- [17] R.N.Wilson, Reflecting Telescope Optics, Springer, Berlin (1996), Vol. I.
- [18] R.N.Wilson, Reflecting Telescope Optics, Springer, Berlin (1999), Vol. II.
- [19] C.G.Wynne, Astrophys. J. 152, 675 (1968).
- [20] C.G.Wynne, *Progress in Optics*, Ed. by E.Wolf, North-Holland, Amsterdam (1972), Vol. 10, p. 139.
- [21] C.G.Wynne, The Observatory **106**, 163 (1986).
- [22] C.G.Wynne and S.P.Worswick, Mon. Not. R. Astron. Soc. 220, 657 (1986).

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